Light by light diffraction in vacuum

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We show that a laser beam can be diffracted by a more concentrated light pulse due to quantum vacuum effects. We compute analytically the intensity pattern in a realistic experimental configuration, and discuss how it can be used to measure for the first time the parameters describing photon-photon scattering in vacuum. In particular, we show that the Quantum Electrodynamics prediction can be detected in a single-shot experiment at future 100 petawatt lasers such as ELI or HIPER. On the other hand, if carried out at one of the present high power facilities, such as OMEGA EP, this proposal can lead either to the discovery of non-standard physics, or to substantially improve the current PVLAS limits on the photon-photon cross section at optical wavelengths. This new example of manipulation of light by light is simpler to realize and more sensitive than existing, alternative proposals, and can also be used to test Born-Infeld theory or to search for axion-like or minicharged particles.

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The linear propagation of light in vacuum, as described by Maxwell equations, is a basic assumption underlying our communication system, allowing e.g. that different electromagnetic waves do not keep memory of their possible crossing in the way to their reception points. However, this superposition principle is expected to be violated by quantum effects. In fact, Quantum Electrodynamics (QED) predicts the existence of Photon-Photon Scattering in Vacuum (PPSV) mediated by virtual charged particles running in loop diagrams [1], although the rate is negligible in all the experiments that have been performed up to now. On the other hand, additional, possibly larger contributions to the process may appear in non-standard models such as Born Infeld theory [2–4] or in new physics scenarios involving minicharged [5] or axion-like [6] particles. Therefore, the search for PPSV is important not only to demonstrate a still unconfirmed, fundamental quantum property of light, but also to either discover or constrain these kinds of new physics.

In the last few years, there has been an increasing interest in the quest for PPSV [7–10]. Here, we present a new scenario to search for this phenomenon using ultra-high power lasers [11]. In our proposal, two almost contra-propagating laser pulses cross each other. Due to PPSV, the more concentrated pulse behaves like a phase object diffracting the wider beam. The resulting intensity pattern can then be observed on a screen, and will correspond to a direct detection of scattered photons.

The effective Lagrangian for photon-photon scattering. Following Ref. [9], we will assume that for optical wavelengths the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ are described by an effective Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_0 + \xi_L \mathcal{L}_0^2 + \frac{7}{4}\xi_T \mathcal{G}^2, \tag{1}$$

being $\mathcal{L}_0 = \frac{\epsilon_0}{2}\left(\mathbf{E}^2 - c^2\mathbf{B}^2\right)$ the Lagrangian density of the linear theory and $\mathcal{G} = \epsilon_0 c(\mathbf{E}\cdot\mathbf{B})$.

In Quantum Electrodynamics, $\mathcal{L}$ would be the Euler-Heisenberg effective Lagrangian [12], that coincides with Eq. (1) with the identification $\xi_L^{QED} = \xi_T^{QED} \equiv \xi$, being

$$\xi = \frac{8\alpha^2\hbar^3}{45 m_e^4 c^5} \simeq 6.7\times 10^{-30}\frac{m^3}{J}. \tag{2}$$

However, in Born-Infeld theory [2–4], or in models involving a new minicharged (or milli-charged) [5] or axion-like [6] particle, ξ_L and ξ_T will have different values, as computed in Ref. [9].

On the other hand, the current 95% C.L. limit on PPSV at optical wavelengths has been obtained by the PVLAS collaboration [8]. As shown in Ref. [9], it can be written as

$$\frac{|7\xi_T - 4\xi_L|}{3} < 3.2\times 10^{-26}\frac{m^3}{J}. \tag{3}$$

Assuming $\xi_L = \xi_T \equiv \xi^{exp}$ as in QED, this can be translated in the limit $\xi^{exp} < 3.2\times 10^{-26} m^3/J$, which is 4.6×10^3 times higher than the QED value of Eq. (2).

Proposal of an experiment: analytical computations. In our present proposal, illustrated in Fig. 1, a polarized ultrahigh power Gaussian pulse A of transverse width w_A crosses an almost contra-propagating polarized 'probe' laser pulse B of width $w_B \gg w_A$. For simplicity, we assume that the two beams have the same mean wavelength $\lambda = 2\pi/k$ and frequency $\nu = c/\lambda = ck/2\pi$, although in principle they may have different durations τ_A and τ_B. We also suppose that the uncertainty in frequency $\Delta\nu$ is much smaller than ν, in such a way that we can consider the pulses as being monochromatic with a good approximation. Similarly, we assume that the uncertainty in the components of the wave vector are much smaller than k.

From Ref. [9], we learn that the central part of the probe B, after crossing the pulse A, acquires a phase shift

$$\phi_{L,T}(0) = I_A(0) k \tau_A a_{L,T} \xi_{L,T} \tag{4}$$

where $I_A(0)$ is the peak intensity of the high power beam at the crossing point, the indexes L and T refer to the two

beams having parallel or orthogonal linear polarizations, respectively, and we have defined $a_L = 4$ and $a_T = 7$.

Let $A_A = A_A(0)\exp\left(-\frac{r^2}{w_A^2}\right)$ and $A_B = A_B(0)\exp\left(-\frac{r^2}{w_B^2}\right)$ describe the dependences of the non-vanishing components of the two waves on the radial coordinate $r \equiv \sqrt{x^2+y^2}$ orthogonal to the direction of the motion, chosen in the z-axis. The intensity of the pulse A in the colliding region will then have the transverse distribution $I_A = I_A(0)\exp\left(-2\frac{r^2}{w_A^2}\right)$. As a consequence, the space-dependent phase shift of the wave B just after the collision with the beam A is

$$\phi(r) = \phi(0)\exp\left(-\frac{2r^2}{w_A^2}\right), \tag{5}$$

where we understand one of the sub-indexes L or T.

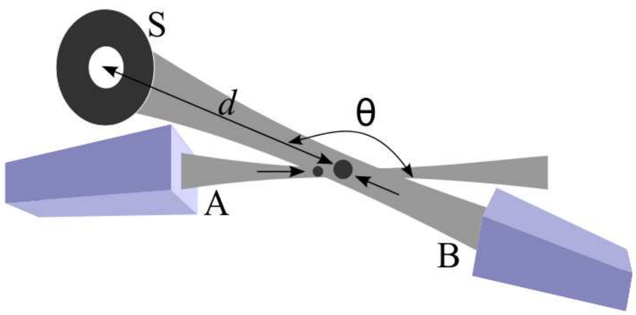

FIG. 1: Sketch of the proposed experiment. An ultra-intense laser pulse A and a wider probe beam B, both moving in a high vacuum, are focused to a region where they collide at an angle θ close to π. The diffracted part of the probe is then observed at a distance d on the ring screen S. (In a minimal version, a single laser can produce both beams.)

Due to this phase shift, the shape of the pulse B becomes $A_B = A_B(0)\exp\left[-\frac{r^2}{w_B^2} + i\phi(r)\right]$. As discussed in Ref. [9], ϕ is expected to be very small at all the facilities that will be available in the near future. Therefore $\exp[i\phi(r)] \simeq 1 + i\phi(r)$ with a very good approximation, and we obtain

$$A_B = A_B(0)\left[\exp\left(-\frac{r^2}{w_B^2}\right) + i\phi(0)\exp\left(-\frac{r^2}{w_0^2}\right)\right], \tag{6}$$

where we have defined $w_0 \equiv (2/w_A^2 + 1/w_B^2)^{-1/2}$.

After the collision, the field A_B propagates linearly, so that we can just sum the free evolution of each term in Eq. (6), that can be computed with the approximation $\omega = c\sqrt{k^2+k_\perp^2} \simeq c(k + k_\perp^2/2k)$ for the angular frequency, where $\mathbf{k}_\perp = (k_x, k_y, 0)$, assuming that $\Delta k_{x,y} = 1/w \ll k$. As a result, the linear evolution of A_B on the screen-detector plane $z = d$ produces an intensity pattern $I(r) = I_U(r) + I_D(r) + I_I(r)$, where I_U and I_D correspond to the undiffracted and diffracted waves respectively, and I_I represents the interference term. The result is

$$I_U(r) = I_B(0)\tfrac{w_B^2}{w_U^2}\exp\left(-\tfrac{2r^2}{w_U^2}\right),$$
$$I_D(r) = I_B(0)\phi(0)^2\tfrac{w_0^2}{w_D^2}\exp\left(-\tfrac{2r^2}{w_D^2}\right), \tag{7}$$

where we have defined the widths of the undiffracted and diffracted patterns, $w_U \equiv w_B\sqrt{1+(2d/kw_B^2)^2}$ and $w_D \equiv w_0\sqrt{1+(2d/kw_0^2)^2}$, and $I_B(0)$ is the peak intensity of the wave B at the collision point, that can be related to the total power $P_B = P_U$ of the pulse B as $P_U = \frac{\pi}{2}w_B^2 I_B(0)$. The interference term $I_I(r)$ can be evaluated by multiplying $2\sqrt{I_U(r)I_D(r)}$ by the factor

$$\sin\left[\frac{d\lambda(w_0^4 - w_B^4)r^2}{\pi w_B^2 w_0^2 w_U^2 w_D^2} + \arctan\left(\frac{d\lambda}{\pi w_0^2}\right) - \arctan\left(\frac{d\lambda}{\pi w_B^2}\right)\right],$$

and turns out to be numerically negligible, as compared to $I_U(r) + I_D(r)$, in all the configurations that we will discuss below.

On the other hand, the total power of the diffracted pulse can be obtained by integrating Eq. (7) in the screen plane, so that $P_D = \frac{\pi}{2}w_0^2\phi(0)^2 I_B(0)$, which is much smaller than P_U. However, an interesting feature of Eq. (7) is that the diffracted wave is distributed in an area of width $w_D \sim 2\sqrt{2}d/kw_A \gg w_U \sim 2d/kw_B$ (for $2d \gg kw_B^2$), so that it can be separated from the undiffracted wave e.g. by making a hole in the screen. Let r_0 be the radius of the central region that is eliminated from the screen. We require that the total power $P_D(r > r_0) = \frac{\pi}{2}w_0^2\phi(0)^2 I_B(0)\exp(-2r_0^2/w_D^2)$ due to the diffracted wave for $r > r_0$ is much larger than the the power of the undiffracted wave in the same region, $P_U(r > r_0) = \frac{\pi}{2}w_B^2 I_B(0)\exp(-2r_0^2/w_U^2)$. A safe choice can be $P_D(r > r_0) = 100 P_U(r > r_0)$, that implies

$$r_0 = w_D w_U \sqrt{\frac{\log\left(\frac{10 w_B}{\phi(0) w_0}\right)}{w_D^2 - w_U^2}}. \tag{8}$$

Finally, using Eqs. (4) and (7), we can compute the number of diffracted photons that will be detected after $\mathcal{N}$ repetitions of the experiment in the ring region $r_0 < r < R$ of the screen, being R its external radius. We obtain

$$N_D^{\mathcal{N}} = \frac{8f\mathcal{N}}{\pi\hbar c}\frac{E_A^2 E_B w_0^2}{\lambda w_A^4 w_B^2}\left(e^{-\frac{2r_0^2}{w_D^2}} - e^{-\frac{2R^2}{w_D^2}}\right)(a\xi)^2_{L,T}, \tag{9}$$

where f is the efficiency of the detector, and $E_A = P_A\tau_A$ and $E_B = P_B\tau_B$ are the total energies of the two pulses.

Angular constraints and Optimization of the sensitivity. Eq. (9) shows that the number of scattered photons is proportional to the product $E_A^2 E_B$ of the energies of the two laser beams. It would then be convenient to use a ultrahigh power pulse also for the probe B. This can be done economically by producing both beams simultaneously, e.g. by dividing a single pulse of energy

$E = E_A + E_B$ before the last focalizations. The maximum value for N_D is then obtained by taking $E_A = 2E/3$ and $E_B = E/3$.

The other parameters that can be adjusted in order to maximize N_D are the widths w_A and w_B of the two colliding beams. The choice of w_A is constrained by the requirement that the pulse A must not spread in a significant way during the crossing, so that $w_A \gtrsim \sqrt{c\tau_B \lambda/\pi}$. However, a more stringent constraint, involving also the angle θ, originates from the condition that the center of pulse A has to remain close to the central part of beam B during the interaction. This implies that $c\tau_B \tan(\pi - \theta) \ll w_A$. A safe choice can then be $c\tau_B \tan(\pi - \theta) = w_A/10$. On the other hand, the angle $\pi - \theta$ has to be such that, out of the collision point, the trajectories of the two beams are separated by a distance sufficiently larger than their width. We conservatively ask that such a distance is 6 times the width $\sim z\lambda/\pi w_A$ of the beam A at the distance z, although one can keep in mind that smaller separations, if they turned out to be experimentally viable, would allow for better sensitivities. For small $\pi - \theta$, we then have $\pi - \theta \simeq 6\lambda/\pi w_A$, and we can solve for w_A,

$$w_A = \sqrt{60 c\tau_B \lambda/\pi}. \tag{10}$$

On the other hand, the value of $w_B > w_A$ that maximizes N_D will be computed numerically, and the outer radius R will be chosen slightly larger than $\sqrt{2} w_D \sim 2\lambda d/\pi w_A$, by requiring that only a few percent of the diffracted wave is lost.

Finally, the measurement of the number of diffracted photons N_D can be used to determine the values of the parameters ξ_L and ξ_T. To evaluate the best possible sensitivity, we will suppose that the background of thermal photons and the dark count of the detectors can be made much smaller than the signal. Although this goal may be difficult in practice, in principle it can be achieved by cooling the ring detector and covering it with a filter that selects a tiny window of wavelengths around λ, and by optically isolating the experimental area within the time of response of the detector, which should be as small as possible (the ultrashort time of propagation of the beams can be neglected in comparison). Of course, the actual background should be measured by performing the control experiments in the absence of the beams, and with only one pulse at a time.

Under these assumptions, the best sensitivity would correspond to the detection of, say, 10 diffracted photons, so that the zero result could be excluded within three standard deviations. The ideal, minimum values of ξ_L and ξ_T that could be measured would then be given by Eq. (9), taking $N_D^{\mathcal{N}} = 10$ and all the choices reviewed above. (In the numerical computations that we present below, we also include a small correction $\sin^4(\theta/2)$ that appears in the expression of $\phi(0)$ as shown in Ref. [9].)

Sensitivity at future 100 Petawatt laser facilities. Let us now study the possibility of performing our proposed experiment, in its economical version discussed above, using a 100 Petawatt laser such as ELI [13] or HIPER [14], that are expected to become operative in few years. In this case, we can use the following values: total power $P = 10^{17} W$, duration $\tau = 30 fs$, energy $E = 3kJ$ and wavelength $\lambda = 800nm$. With these data, using Eq. (10), we can compute the suggested value $w_A = 12\mu m$ for the width of the spot to which the pulse A should be focalized at the collision point. Taking e.g. $d = 1m \gg w_B^2/\pi\lambda$, we find numerically that the best choice for w_B is $w_B \simeq 5w_A = 59\mu m$. We then obtain the value $r_0 = 2.1cm$ for the central hole in the screen, $R = 4.8cm$ for its outer radius, and $w_U = 0.43cm$ and $w_D = 3.1cm$ for the widths of undiffracted and diffracted waves. The focused intensities of the two beams are $I_A(0) = 3.1 \times 10^{22} W/cm^2$ and $I_B(0) = 6.2 \times 10^{20} W/cm^2$, and the angle $\pi - \theta = 0.13 rad$. Even if we assume an efficiency as small as $f = 0.5$, which is a realistic value today for $\lambda \sim 800nm$, we obtain that our proposed experiment can resolve ξ_L and ξ_T as small as $\xi_L^{\rm limit} = 2.8 \times 10^{-30} \mathcal{N}^{-1/2} m^3/J$ and $\xi_T^{\rm limit} = 1.6 \times 10^{-30} \mathcal{N}^{-1/2} m^3/J$, values that are well below the QED prediction even for $\mathcal{N} = 1$. Therefore, ELI and HIPER will be able to detect PPSV at the level predicted by QED in a single shot experiment. Two single-shot experiments, using parallel and orthogonal polarizations of the colliding waves respectively, would allow to measure both ξ_L and ξ_T.

We can restate this result in terms of the number of diffracted photons per pulse that will be be scattered in the ring detector. In the experiment with orthogonal polarizations, this number is ~ 340, which is two orders of magnitude higher that the value obtained in the scenario of Ref. [10]. This indicates that our present proposal is by far more sensitive than that of Ref. [10], in spite of the fact that we have applied much more realistic assumptions on the width w_A of the higher power pulse in order to clearly separate the beams out of the crossing region. Moreover, the configuration of Ref. [10] is less economical, since it requires an additional high power laser, and it will also present a greater experimental difficulty, as far as it needs the alignment of three ultrashort pulses (two of them having a spot radius as small as $0.8\mu m \sim \lambda$).

Let us now compare our present proposal with that of Ref. [9]. The main differences between the two scenarios, both based in the crossing of two contra-propagating beams, are the following: 1) in Ref. [9], the probe pulse could have much smaller power; 2) in Ref. [9], the two beams had the same width, and the effect of PPSV was observed by measuring the phase shift of the probe by comparing it with a third beam that was originally in phase with it. Actually, the sensitivity calculated in Ref. [9] corresponded to a greater power, $P \sim 10^{18} W$, and shorter $\tau \sim 10^{-14} s$, giving $w \sim 3\mu m$ for $I \sim 10^{25} W cm^{-2}$. To compare that result with the present proposal, we have to use the same power, $P \sim 10^{17} W$, and restate the sensitivity obtained in a single shot in Ref. [9] as $\xi_L^{\rm lim} = 2.8 \times 10^{-8} w^2 \lambda/(16E)$ and $\xi_T^{\rm lim} = 2.8 \times 10^{-8} w^2 \lambda/(28E)$, where $E = P\tau$ and w

are the total energy and focused width of the high power beam. It can be seen that a safe choice for the angle θ in that configuration would have needed $w \simeq c\tau$, so that even at $P = 10^{17}W$ the setup of Ref. [9] would have preferred to use $\tau \sim 10fs$ instead of $30fs$. This would lead to $\xi_L^{\rm lim} \simeq 1.3 \times 10^{-29} m^3/J$ and $\xi_T^{\rm lim} \simeq 7.2 \times 10^{-30} m^3/J$. These results would be significantly worse than those that we have obtained above in the present proposal. Moreover, an additional advantage of our new configuration is the fact that it can be systematically improved by increasing the number $\mathcal{N}$ of crossing events in Eq. (9).

Sensitivity at present Petawatt lasers. Facilities such as OMEGA EP [15] can already provide $P = 10^{15}W$, $\tau = 1ps$ and $E = 1kJ$ at $\lambda = 1053nm$. From Eq. (10), we obtain $w_A = 78\mu m$. Taking $d = 1m$, we find numerically the optimal choice $w_B = 5w_A = 0.39mm$, leading to $r_0 = 5.2mm$, $R = 10mm$, $w_U = 0.95mm$, $w_D = 6.2mm$, $I_A(0) = 7.0 \times 10^{18} W/cm^2$, $I_B(0) = 1.4 \times 10^{17} W/cm^2$ and $\pi - \theta = 0.026rad$. Unfortunately, at present the efficiency of photon detectors for $\lambda \sim 1\mu m$ is just $f \simeq 0.1$. Nevertheless, we obtain that our proposed experiment can resolve ξ_L and ξ_T as small as $\xi_L^{\rm limit} = 2.1 \times 10^{-27} \mathcal{N}^{-1/2} m^3/J$ and $\xi_T^{\rm limit} = 1.2 \times 10^{-27} \mathcal{N}^{-1/2} m^3/J$. Even for the single shot experiment, $\mathcal{N} = 1$, this result is at least an order of magnitude below the current limit of Eq. (3). As a result, this experiment at present facilities can already either detect PPSV of non-standard origin, or substantially improve the limits on ξ_L and ξ_T. In the former case, the measurement of both ξ_L and ξ_T can be used to discriminate between different kind of new physics, such as Born Infeld theory [2–4] or models involving minicharged [5] or axion-like [6] particles, using the expressions for the corresponding contributions that have been computed in Ref. [9].

Finally, we note that in the minimal realization of our proposal, using only one high energy laser pulse divided in two parts of the same time duration $\tau_A = \tau_B = \tau$, the optimal w_B turns out to be proportional to w_A (usually by a factor ~ 5). Taking into account Eqs. (9) and (10), this implies that the discovery potential $N_D^{\mathcal{N}}$ for PPSV is proportional to $fP^3\tau/\lambda^3$. One could then study the possibility of using a $10PW$ laser having a wavelength in the visible window, in order to compensate in part the lower power (as compared with the $100PW$ case discussed above) with the higher f and λ^{-3} factors. In this case, assuming $\tau \sim 30fm$, our computations show that PPSV at the QED rate can be detected by accumulating a number of repetitions $\mathcal{N} \sim 10$.

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[1] V. Costantini, B. De Tollis, G. Pistoni, Nuovo Cimento **2A**, 733 (1971).

[2] M. Born, Proc. R. Soc. London, **143**, 410 (1934); M. Born and L. Infeld, Proc. R. Soc. London, **144**, 425 (1934).

[3] V. I. Denisov , I. V. Krivchenkov, and N. V. Kravtsov, Phys. Rev. D **69**, 066008 (2004).

[4] P. Gaete and J. Helayel-Neto, arXiv:0912.2321 (2009).

[5] B. Holdom, Phys. Lett. B **166**, 196 (1986); B. Holdom, Phys. Lett. B **178**, 65 (1986); E. Golowich, R.W. Robinett, Phys. Rev. D **35**, 391 (1987); M. I. Dobroliubov and A. Y. Ignatiev, Phys. Rev. Lett. **65**, 679 (1990); S. Davidson, B. Campbell and D. Bailey, Phys. Rev. D **43**, 2314 (1991); S. Davidson, S. Hannestad, G. Raffelt, J. High Energy Phys. **05(2005)**, 003 (2005); H. Gies, J. Jaeckel, A. Ringwald, Phys. Rev. Lett. **97**, 140402 (2006); M. Ahlers, H. Gies, J. Jaeckel, A. Ringwald, Phys. Rev. D **75**, 035011 (2007).

[6] E. Masso and R. Toldra, Phys. Rev. D **52**, 1755 (1995); J. A. Grifols, E. Masso and R. Toldra, Phys. Rev. Lett. **77**, 2372 (1996); J. W. Brockway, E. D. Carlson and G. C. Raffelt, Phys. Lett. B **383**, 439 (1996); E. Masso and R. Toldra, Phys. Rev. D **55**, 7967 (1997); E. Masso, Nucl. Phys. Proc. Supp. **114**, 67 (2003); P. Arias, J. Gamboa, H. Falomir and F. Méndez, Mod. Phys. Lett. A **24**, 1289 (2009).

[7] S. L. Adler, Ann. Phys. **67**, 599 (1971); E. B. Aleksandrov, A. A. Anselm, and A. N. Moskalev, Zh. Eksp. Teor. Fiz. **89**, 1181 (1985)[Sov. Phys. JETP **62**, 680 (1985)]; Y. J. Ding and A. E. Kaplan, Phys. Rev. Lett. **63**, 2725 (1989); F. Moulin and D. Bernard, Opt. Comm. **164**, 137 (1999); D. Bernard *et al.*, Eur. Phys. J. D **10**, 141 (2000); G. Brodin, M. Marklund, and L. Stenflo, Phys. Rev. Lett. **87**, 171801 (2001); G. Brodin, et al. Phys. Lett. A **306**, 206 (2003); E. Lundstrom et al., Phys. Rev. Lett. **96**, 083602 (2006); T. Heinzl et al., Opt. Comm. **267**, 318 (2006); A. Di Piazza, K.Z. Hatsagortsyan, C.H. Keitel, Phys. Rev. Lett. **97**, 083603 (2006); M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006); A. Ferrando, H. Michinel, M. Seco, and D. Tommasini, Phys. Rev. Lett., **99**, 150404 (2007); D. Tommasini, A. Ferrando, H. Michinel, M. Seco, Phys. Rev. A **77**, 042101 (2008); F. Ehlotzky *et al.*, Rep. Prog. Phys. **72**, 046401 (2009); H. Gies, Eur. Phys. J. D **55**, 311 (2009); M. Marklund and J. Lundin, Eur. Phys. J. D **55**, 319 (2009); S. V. Bulanov *et al.*, Eur. Phys. J. D **55**, 483 (2009); G. Zavattini and E. Calloni, Eur. Phys. J. C **62**, 459 (2009).

[8] M.Bregant et al., PVLAS, Phys. Rev. D **78**, 032006 (2008).

[9] D. Tommasini, A. Ferrando, H. Michinel, M. Seco, J. High Energy Phys. **11(2009)**, 043 (2009).

[10] B. King, A. Di Piazza and C. H. Keitel, Nature Photonics **4**, 92 (2010); M. Marklund, Nature Photonics **4**, 72 (2010).

[11] G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. **78**, 310 (2006).

[12] W. Heisenberg and H. Euler, Z. Physik **98**, 714 (1936).

[13] http://www.extreme-light-infraestructure.eu.

[14] http://www.hiper-laser.org.

[15] http://www.lle.rochester.edu/05_omegalaserfacility.